Functional Analysis, Mathematical Physics, and Dynamical Systems

(FAMPDS)

Joint American-Ukrainian Virtual Colloquium Series

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Talk 8: *Can One Hear the Shape of a Fractal Drum?*

Michel L. Lapidus (UC, Riverside)

Abstract

A well-known problem in mathematics and physics consists in understanding how the geometry (or shape) of a musical instrument affects it sound. This gives rise to two related types of mathematical problems: direct spectral problems (how the shape of a drum affects its sound) and inverse spectral problems (how one can recover the shape of a drum from its sound). Here, we consider both types of problems in the context of drums with fractal (that is, very rough) boundary. We show, in particular, that one can "hear" the fractal dimension of the boundary (a certain measure of its roughness) and, in certain cases, a fractal analog of its length. In the special case of vibrating fractal strings (the one-dimensional situation), we show that the corresponding inverse spectral problem is intimately connected with the Riemann Hypothesis, which is arguably the most famous open problem in mathematics and whose solution will likely unlock deep secrets about the prime numbers. In conclusion, we briefly explain how this work eventually gave rise to a mathematical theory of complex fractal dimensions (developed by the author and his collaborators), which captures the vibrations that are intrinsic to both fractal geometries and the prime numbers.

Friday, April 30, 10:00-11:00 AM (PDT), 20:00-21:00 (EET)

Online via Zoom at
https://fresnostate.zoom.us/j/5233106532