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Chernoff approximation of operator semigroups generated by Markov processes

Abstract

We present a method to approximate operator semigroups generated by Markov processes and, therefore, transition probabilities of these processes. This method is based on the Chernoff theorem. In some cases, Chernoff approximations provide also discrete time Markov processes approximating the considered (continuous time) processes (in particular, Euler-Maruyama Schemes for the related SDEs). In some cases, Chernoff approximations have the form of limits of n iterated integrals of elementary functions as $n \to \infty$ (in this case, they are called Feynman formulae) and can be used for direct computations and simulations of Markov processes. The limits in Feynman formulae sometimes coincide with (or give rise to) path integrals with respect to probability measures (such path integrals are usually called Feynman-Kac formulae). Therefore, Feynman formulae can be used to approximate the corresponding path integrals and to establish relations between different path integrals.

In this talk, we discuss Chernoff approximations for (semigroups generated by) Feller processes in $\mathbb{R}^d$. We are also interested in constructing Chernoff approximations for Markov processes which are obtained by different operations from some original Markov processes, assuming that Chernoff approximations for the original processes are already known. In this talk, we present Chernoff approximations for such operations as: a random time change via an additive functional of a process, a subordination (i.e., a random time change via an independent a.s. nondecreasing 1-dim. Lévy process), killing of a process upon leaving a given domain, reflecting of a process. These results allow, in particular, to obtain Chernoff approximations for subordinate diffusions on strar graphs and compact Riemannian manifolds. Moreover, Chernoff approximations can be further used to approximate solutions of some time-fractional evolution equations and hence to approximate marginal densities of the corresponding non-Markovian stochastic processes.