

An introduction to \mathcal{D} -module theory

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October 2025

I will first explain why a system of linear equations on a ring A is nothing but a finitely presented A -module M .

Then I will study the sheaf of rings \mathcal{D}_X of holomorphic differential operators on a complex manifold X . I will formulate the Cauchy problem in this setting and generalize the classical Cauchy-Kowalevski theorem to \mathcal{D}_X -modules, following Kashiwara's thesis.

Finally, I will have a glance to hyperfunction solutions of \mathcal{D}_X -modules.

References

- [Kas03] Masaki Kashiwara, *\mathcal{D} -modules and microlocal calculus*, Translations of Mathematical Monographs, vol. 217, 2003.
- [KS90] Masaki Kashiwara and Pierre Schapira, *Sheaves on manifolds*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 292, Springer-Verlag, Berlin, 1990.