A law of the iterated logarithm for small counts in the infinite balls-in-boxes scheme

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The infinite balls-in-boxes scheme is defined as follows. Balls are thrown independently into an infinite array of boxes numbered 1, 2, ..., with a probability p_k of hitting the box k. Here, $(p_k)_{k\in\mathbb{N}}$ is a discrete probability distribution with infinitely many $p_k > 0$.

We are interested in the number of boxes containing exactly and at least j balls after n balls were distributed. Denote these quantities via $\mathcal{K}_j^*(n)$ and $\mathcal{K}_j(n)$, respectively. With j fixed, $\mathcal{K}_j^*(n)$ are called *small counts*.

In [1], we proved a law of the iterated logarithm (LIL) for $\mathcal{K}_j(n)$ as $n \to \infty$. Its proof exploits a Poissonization technique and is based on a new LIL for infinite sums of independent indicators $\sum_{k\geq 1} \mathbb{1}_{A_k(t)}$ as $t \to \infty$, where the family of events $(A_k(t))_{t\geq 0}$ is nondecreasing in t.

The crucial difference between $\mathcal{K}_j(n)$ and $\mathcal{K}_j^*(n)$ is that the former process in nondecreasing in n while the latter is not. Therefore, application of the aforementioned LIL to *small counts* is impossible.

I will discuss how we lifted the monotonicity assumption in [2] and obtained the extended LIL for infinite sums of independent indicators $\sum_{k\geq 1} \mathbb{1}_{A_k(t)}$ as $t \to \infty$, where the family of events $(A_k(t))_{t\geq 0}$ is ***not necessarily monotone*** in t. As a corollary, we state LIL for the small counts as the number of balls thrown becomes large.

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