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Physical origin of the fractional Brownian motion and related Gaussian processes arising in the models of anomalous diffusion

(joint work with Christian Bender, Mirko D'Ovidio and Gianni Pagnini)

Experimentally well-established, anomalous diffusion (AD) is a phenomenon observed in many different natural systems belonging to different research fields. In particular, AD has become fundational in living systems after a large use of single-particle tracking techniques in the recent years. Generally speaking, AD labels all those diffusive processes that are governed by laws that differ from that of classical diffusion, namely, all that cases when particles' displacements do not accomodate to the Gaussian density function and/or the variance of such displacements does not grow linearly in time.

We propose an attempt for establishing the physical origin of AD within the classical picture of a test-particle kicked by infinitely many surrounding particles. We consider a stochastic dynamical system where the microscopic thermal bath is the source for the mesoscopic Brownian motion of a bunch of N particles that express the environment of a single test-particle. Physical conservation principles, namely the conservation of momentum and the conservation of energy, are met in the considered particle system in the form of the fluctuation-dissipation theorem for the motion of the surround-particles. The key feature of the considered particle-system is the distribution of the masses of the particles that compose the surround of the test-particle. When the number of mesoscopic Brownian particles N is large enough for providing a crowded environment, then the test-particle displays AD characterised by the distribution of the masses of the surround-particles. More precise, we prove that, in the limit $N \to \infty$, the test-particle diffuses according to a quite general non-Markovian Gaussian process $(Z_t)_{t\geq 0}$ characterised by a covariance function

$$\mathrm{Cov}(Z_t, Z_s) = C(v(t) + v(s) - v(|t-s|)), \qquad (1)$$

where $v(\cdot)$ is determined by the distribution of the masses of the surround-particles. With a particular choice of distribution of surround-particles, we obtain fractional Brownian motion (fBm) with Hurst parameter $H \in (1/2, 1)$ as a special case. In this respect, we remind that the fBm experimentally turned out to be the underlying stochastic motion in many living systems. We present also some distributions of masses of the surround-particles which lead to a mixture of independent fractional Brownian motions with diferent Hurst parameters or to a classical Wiener process as a limiting process $(Z_t)_{t\geq 0}$. Moreover, we present some distributions of masses of the surround-particles leading to the limiting processes which perform a transition from ballistic diffusion to superdiffusion, or from ballistic diffusion to classical diffusion.

Furthermore, the constant C in formula (1) depends on the coupling parameter between the test-particle and the surround. Therefore, if we consider several independent identical copies of the same Brownian surround and immerse into each copy of the surround a single test particle of the same art but with its own individual characteristics (assuming our test-particle is a complex macromolecule with its individual shape, radius, densty e.t.c.), we may obtain different coupling parameters and hence different coefficients C in the covariance of the limiting Gaussian process $(Z_t)_{t\geq 0}$ in different copies of the experiment. This fact may serve as a physical basis for the formulation of AD within the framework of the superstatistical fBm, where further randomness is provided by a distribution of the diffusion coeffients associated to each diffusing test-particle and also within the framework of its generalisation called diffusing-diffusivity approach, where the diffusion coefficient of each test-particle is no longer a random variable but a process.