Quadratic Stochastic Processes with a Continuous Set of States

Ganikhodjaev, Nasir

V.I. Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan
nasirgani@yandex.ru

Let (E, F) be a measurable space and M be a collection of all probability measures on (E, F).
The family of transition functions

$$\{P(s; x; y; t; A) : x, y \in E, A \in F, s, t \in R_+, t - s \geq 1\}$$

is called *Quadratic Stochastic Process* [1], if it satisfies the following conditions:
i)$P(s; x; y; t; A) = P(s, y, x, t, A)$ for all $x, y \in E,\ A \in F$;
ii)$P(s; x; y; t; \cdot) \in M$ for all $x, y \in E$;
iii) for any fixed $A \in F$ the function $P(s; x; y; t; A)$ as function of two variables x, y is measurable.
Unlike Markov processes there are two different forms of Kolmogorov-Chapmen equations:
for an initial measure $m_0 \in M$ and arbitrary $s, \tau, t \in R_+$; such that $t - \tau \geq 1$ and $\tau - s \geq 1$, we have

$$P(s, x, y, t, A) = \int_E \int_E P(s, x, y, \tau, du) P(\tau, u, v, t, A) m_\tau(dv) \quad (1)$$

or

$$P(s, x, y, t, A) = \int_E \int_E \int_E \int_E P(s, x, z, \tau, du) P(s, x, y, \tau, dw) P(\tau, u, w, t, A) m_s(dv) \quad (2)$$

Equations (1) and (2) can be interpreted as different rules for the appearance of the "grandchildren". These equations also have implications for chemical treatments. So, the appearance of particles in reactions occurring in ordinary chemical kinetics are described either by the equation of type (1) or by the equation of type (2) which reflect the appearance of particles in processes of catalysis.
The goal of this presentation is to define *Brownian* quadratic stochastic process.

References:

1. N. Ganikhodjaev. On stochastic processes generated by quadratic operators. *Journal Theor. Prob.* , **4** (**1991**) 639–653.