Non-ergodic quadratic stochastic operators with countable state space

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Let

$$S^N = \{\mathbf{x} = (x_i)_{i=1}^{\infty} : \forall i \ \ x_i \geq 0, \sum_{i=1}^{\infty} x_i = 1\} \tag{1}$$

is the set of all probability measures on $(N, \mathcal{P}(N))$, where N is the set of positive integers. In this presentation we consider the limit behaviour of the trajectories of the following Volterra quadratic stochastic operators $V : S^N \to S^N$ with countable state space N as follows.

$$\begin{aligned}
&(V\mathbf{x})_1 = x_1[1 + ax_2 - ax_3 + ax_4 - ax_5 + ax_6 - ax_7 + \cdots] \\
&(V\mathbf{x})_2 = x_2[1 - ax_1 + ax_3 - ax_4 + ax_5 - ax_6 + ax_7 - \cdots] \\
&\ldots \\
&\ldots \\
&(V\mathbf{x})_{2n-1} = x_{2n-1}[1 + ax_1 - ax_2 + \cdots - ax_{2n-2} + ax_{2n} - ax_{2n+1} + ax_{2n+2} - ax_{2n+3} + \cdots] \\
&(V\mathbf{x})_{2n} = x_{2n}[1 - ax_1 + ax_2 - \cdots - ax_{2n-1} + ax_{2n+1} - ax_{2n+2} + \cdots] \\
&\ldots \\
&\ldots
\end{aligned} \tag{2}$$

where $a \in [-1, 1]$.
In 1978, Zakharevich [2] proved that the following Volterra operator on S^2

$$\begin{aligned}
x_1' &= x_1(1 + x_2 - x_3) \\
x_2' &= x_2(1 - x_1 + x_3) \\
x_3' &= x_3(1 + x_1 - x_2)
\end{aligned} \tag{3}$$

is a non-ergodic transformation. The transformation (2) is the generalization of Zakharevich's example (3). We will discuss the ergodic properties of the transformations (2).

Список литературы

[1] Ulam, S. *A collection of mathematical problems*, Interscience Publishers,New-York-London 1960.

[2] Zakharevich M.I. On behavior of trajectories and the ergodic hypothesis for quadratic transformations of the simplex,*Russian Math.Surveys* **33**, 265-266 (1978)