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Modern problems of the theory of stochastic processes and their applications

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Abstracts

Contents

Copula functions and its applications in survival analysis with covariates

A.A. Abdushukurov, R.S. Muradov [1]

Copula (dependence) functions which connect the marginal distributions to their joint distributions, are useful in simulating the linear or nonlinear relationships among multivariate statistical data in the survival analysis studies. Copula is a multivariate distribution function with marginally uniform random variables on $[0,1]$. Now a days, copulas have been applied in statistics, insurance, finance, economics, survival analysis, image processing, and engineering applications.

The problem of estimating of jointly survival function from incomplete data has been considered by authors [1-2]. In the special bivariate case, there are numerous examples of paired data representing the times to death of individuals (married couples or twins), the failure times of components of system and others which subject to random censoring. At present time, there are several approaches to estimating of survival functions of vectors of lifetimes. Moreover, the random variables (r.v.-s) of interest (lifetimes) and censoring r.v.-s can be also influenced by other variable, often called prognostic factor or covariate. In medicine, dose a drug and in engineering some environmental conditions (temperature, pressure) are influenced to the observed variables. The basic problem consist in estimation of jointly distribution of lifetimes by such censored dependent data with used copula (dependence) functions. The aim of paper is considering this problem in the case of right random censoring model in the presence of covariate.

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Hausdorff dimension of invariant measure of piecewise linear circle maps with two break points

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In this paper we study pointwise and Hausdorff dimensions of invariant measures for circle diffeomorphisms. The notion of pointwise (or local) dimension was introduced by Young in [1]. It plays an important role in dimension theory of dynamical systems.

Let μ be be a probability measure on a metric space X. Lower and upper pointwise dimensions at a point x are defined as:

$$\underline{d}_\mu(x) = \liminf_{r\to 0} \frac{\log \mu(B(x,r))}{\log r} \text{ and } \overline{d}_\mu(x) = \limsup_{r\to 0} \frac{\log \mu(B(x,r))}{\log r}$$

where $B(x,r)$ is a ball of radius r centered at x. If the two limits coincide, then their common value $d_\mu(x)$ is called the pointwise dimension of μ at x.

It is well known that if rotation number of circle homeomorphism is rational, it may preserve many measures with different properties, but any ergodic invariant measure is a uniform δ-measure on a periodic orbit. Then Hausdorf dimension of invariant measure equal to zero.

Although a circle homeomorphism with irrational rotation number has uniquely ergodic measure. In this case, it is more complicated to calculate Hausdorff measure or to study rigidity properties. So, we need to study different types of irrational rotation numbers.

Some irrational numbers differ how well they can be approximated by rational numbers. For example, the numbers that can be rapidly approximated by rational numbers are called Liouville otherwise they are called Diophantine. It follows from the work of Herman [2] that sufficiently-smooth circle diffeomorphisms with Diophantine rotation numbers are smoothly conjugate to the linear rotation $T_\rho x = x+\rho$. K.M. Khanin and Ya.G. Sinai in [3] generelated this result for $C^{2+\alpha}, \alpha > 0$ smooth circle maps. Hence, the Hausdorff dimension of their unique invariant measure equals to 1. For any $\beta \in [0,1]$ V.Sadovskaya in [4], constructed C^∞ -smooth circle homeomorphisms whose rotation numbers are Liouville and Hausdorff dimensions equal to β.

K.Khanin and S.Kocić in [5] studied the Hausdorff dimension of circle maps with a break point. They proved that if rotation number $\rho_T \in S$, then Hausdorff dimension of $T-$ invariant measure $\mu : \mu_T$ equal to zero i.e. $\dim_H \mu = 0$.

The class of piecewise linear circle maps with two breaks first were studied by M.Herman in [2]. A.Aliyev proved in [6] that this statement is also correct for the case T is piecewise linear circle homeomorphism with irrational rotation number $\rho_T \in S$ and two break points b_1 and b_2 on different orbits which $\mu([b_1, b_2]) \in G$. Where G is a full set w.r.t. Lebesgue measure on [0,1].

Now we formulate our main theorem.

For each $\beta \in [0,1]$, there exist G_β such that $G_0 = G$ and $\bigcup\limits_{\beta} G_\beta = [0,1]$.

Theorem 1. *Let T be piecewise linear circle homeomorphism with unbounded type irrational rotation number ρ_T and two break points b_1 and b_2 on different orbits which $\mu([b_1, b_2]) \in G_\beta$. Then pointwise dimension $d_\mu(x)$ of μ_T equals to β for μ_T- almost every x.*

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On inequalities for the probability of ruin

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Let $\xi(t)$, $t \geq 0$, $\xi(0) = 0$, be a homogeneous stochastic process with independent increments whose sample functions are continuous on the right. For it we have $E exp\{\lambda\xi(t)\} = exp\{t\psi(\lambda)\}$, $\psi(\lambda) = \gamma\lambda + \frac{\sigma^2\lambda^2}{2} + \int\limits_{-\infty}^{\infty}\left(e^{\lambda x} - 1 - \frac{\lambda x}{1+x^2}\right)dS(x)$, with standard conditions on γ, σ and $S(x)$. For arbitrary ones $a > 0$, $b > 0$, we introduce a random variable , which equal to the moment of the process first exits $\xi(t)$ from the interval $(-a,\ b)$:

$$T = T(a,\ b) = inf\{t \geq 0 :\ \xi(t) \notin (-a,\ b)\}.$$

It is known that a random variable T is finite with probability one if $P(\xi(1) = 0) < 1$. Let $\alpha(a,\ b) = P(\xi(T) \leq -a)$, $\beta(a,\ b) = P(\xi(T) \geq b)$. These quantities are usually called ruin probabilities by analogy with the probabilistic model of a two-player game with discrete time, and it is obvious that $\alpha(a,\ b) + \beta(a,\ b) = 1$.

The exact calculation of ruin probabilities is available only in some particular situations, both in discrete and continuous time. Therefore, the main attention in the study of these quantities began to be paid to asymptotic approaches. Along with asymptotic formulas, the task of obtaining two-sided estimates for the probability of ruin is relevant. They are a natural addition to the asymptotic results. The problem of obtaining two-sided estimates for the probability of ruin for random walks generated by sums of independent identically distributed random variables is considered in [1], [2]. Some two-sided inequalities for the probability of ruin were also obtained in the case of a homogeneous process with independent increments and continuous time. Inequalities for the probabilities of ruin for the case $E\xi(1) < 0$ under various restrictions on the distribution of the process were established in [3]. In point 1 of [4], some lower and upper estimates were proved for the case $E\xi(1) = 0$.

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Our goal is to obtain new two-sided estimates for the probabilities of ruin in the case of $E\xi\left(1\right)=0$. Let us introduce the following notation:

$$l(s)=:\frac{2(s+2)}{s+1}\cdot\frac{a_{s+2}}{a_2},\quad a_s=\int\limits_{-\infty}^{\infty}|x|^s dS\left(x\right),\ \overline{\xi}(t)=\sup_{0\le s\le t}\xi(s),\overline{\overline{\xi}}(t)=\inf_{0\le s\le t}\xi(s),$$

$$R_t(dv,dz)=\int\limits_0^t P(\overline{\xi}(t-s)\in dv)d_sP(\overline{\overline{\xi}}(s)\in dz),$$

$$m_+(t)=:\int\limits_0^\infty\int\limits_0^b\int\limits_{-\infty}^0\left|S(b-z+u-y)\right|R_t(dz,dy)du,$$

$$m_-(t)=:\int\limits_0^\infty\int\limits_0^a\int\limits_{-\infty}^0\left|S(-a+z-u+y)\right|R_t(dy,dz)du,\ \ C(\delta):=2(l(1+\delta))^{1/_{1+\delta}}.$$

Theorem 1. *Suppose that $E\xi\left(1\right)=0$ and for some $\delta>0$ $|\xi\left(1\right)|^{3+\delta}<\infty$. Then for any $t\geq 0$*

$$\frac{a+m_-(t)-C(\delta)}{a+b}\leq\beta\left(a,\ b\right)\leq\frac{a-m_+(t)+C(\delta)}{a+b}.$$

It immediately follows that for any $t\geq 0$

$$\frac{b+m_+(t)-C(\delta)}{a+b}\leq\alpha(a,b)=1-\beta\left(a,\ b\right)\leq\frac{b-m_-(t)+C(\delta)}{a+b}.$$

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Asymptotic behaviour of branching processes with non-homogeneous immigration

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Let μ_n be a number of particles of the Galton-Watson (G-W) branching process at the moment n ($n = 0,\ 1,\ .\ .\ .\ ,\ \mu_0 = 1$) with the generating function (g.f.)

$$F(x) = \sum_{j=0}^{\infty} p_j x^j, \quad p_j = P\left\{\mu_1 = j\right\}, \quad j = 0, 1, ..., \quad |x| \leq 1.$$

The zero state is absorbing for the process μ_n , that is, if $\mu_N = 0$ for some $N > 0$, then $\mu_n = 0$ for all $n > N$. In [1] J.H.Foster considered G-W process modified to allow immigration of particles whenever the number of particles is zero. If $\mu_n = 0$, then, at the moment n, ξ_n particles immigrate to the population, where the number of particles evolves by the law of the G-W process with g.f. $F(x)$.

The asymptotic behavior of branching processes with state-dependent immigration were studied by many authors (see [1]–[3]).

We consider the case when immigration takes place as $\mu\ _n$ =k, $0 \leq k \leq m$, where m is some nonnegative integer. Assume that the intensity of the immigration decreases tending to 0, when the number of descendents increases. Limit theorems for such processes have been studied in [4],[5],[6].

Thus, the immigration is given with g.f.

$$g_{k,n}(x) = \sum_{j=0}^{\infty} q_{kj}(n) x^j, \quad |x| \leq 1, \quad k = 0, 1, ..., m, \quad q_{kj}(n) \geq 0,$$

$$\sum_{j=0}^{\infty} q_{kj}(n) = 1, \qquad n = 0, 1, 2, ... \quad .$$

Let$\{Z_n;\, n = 0, 1, ...\}$ be a number of particles of this process at the moment n. Suppose, that

$$F(x) = x + (1-x)^{1+\nu} L(1-x)$$

where $0 < \nu$? 1 and $L(x)$ is a slowly varying function (s.v.f.) as x ¿ 0.

Introduce the function

$$M(n) = \sum_{k=1}^{n} \frac{N(k)}{k^{1/\nu}},$$

where $N(x)$ is a s.v.f. as x ¿ ? such that

$$\nu N^{\nu}(x) L(x^{-1/\nu} N(x)) \to 1.$$

Denote

$$\alpha_n = \max_{0\le k\le m} g'_{k,n}(1) \qquad \beta_n = \max_{0\le k\le n} g''_{k,n}(1),$$

$$Q_1(n) = \alpha_n \sum_{k=0}^{n} (1 - F_k(0)), \qquad Q_2(n) = (1 - F_n(0)) \sum_{k=0}^{n} \alpha_k$$

where $F_0(x) = x, \quad F_{n+1}(x) = F\left(F_n(x)\right)$.

We suppose that

$$\mathop{Sup}_n \alpha_n < \infty\,, \qquad \mathop{Sup}_n \beta_n < \infty\,,$$

$$0 < \alpha_n \to 0\,, \qquad \beta_n \to 0\,, \qquad n \to \infty.$$

We consider the case $\nu = 1, \quad M(n) \to \infty$ as $n \to \infty$.

Theorem 1. *Let* $\alpha_n \sim \frac{l(n)}{n^r}$, $\beta_n = o\left(Q_1(n)\right), \quad n \to \infty$, *where* $0 \le r \le 1$ *and* $l(n)$ *is a s.v.f. as* $\quad n \to \infty$. *Then*

a) if $r = 0, \quad Q_1(n) \to \theta$ *as* $\quad n \to \infty$ *and* $0 < \theta < 1$, *that*

$$\lim_{n\to\infty} P\{Z_n > 0\} = \frac{\theta}{1+\theta}, \quad EZ_n \sim \frac{n}{M(n)}, \quad n \to \infty.$$

b) if $r = 0, \quad Q_1(n) \to 0, \quad n \to \infty$ *or* $0 < r < 1$, *that*

$$P\{Z_n > 0\} \sim Q_1(n), \qquad EZ_n \sim \frac{n\alpha_n}{1-r}, \quad n \to \infty.$$

c) if $r = 1$ *and* $\beta_n = o\left(Q_1(n) + Q_2(n)\right)$ *as* $\quad n \to \infty$, *that*

$$P\{Z_n > 0\} \sim Q_1(n) + Q_2(n), \qquad EZ_n \sim M(n), \quad n \to \infty.$$

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Equations with interaction, second quantization, stability and polymer models

A. A. Dorogovtsev

In the talk we present an overview of the recent results on stationary behaviour of solutions to equations with interaction. Such equations were introduced by author in 2000 [1] for description of the motion of the large systems of particles where the instant increments of trajectory of fixed particle depend on mass distribution of the whole system. The equation with interaction has the form

$$dx(u,t) = a(x(u,t),\mu_t)dt + \int_{\mathbb{R}}^{d} b(x(u,t),\mu_t,p)W(dp,dt), \tag{1}$$

$$x(u,0) = u, \ u \in \mathbb{R}^d, \ \mu_t = \mu_0 \circ x(\cdot,t)^{-1}.$$

Here W is a Gaussian white noise in $L_2(\mathbb{R}^d \times [0;+\infty])$ responsible for the influence of random media. $x(u,t), t \geq 0$ is the trajectory of the particle which starts from the point $u \in \mathbb{R}^d$. μ_t is the mass distribution of the whole system. Its presence in the coefficients a and b reflects the interaction between particles.

In the first part of the talk we discuss the possible conditions under which the equation with interaction has a stationary solution. It turns out that either such solution is of poor structure [2] or only shift-compactness can be guaranteed [3]. This means that one can expect the stationarity only for certain functionals from the solution. To investigate more detaily what kind of stationary behaviour can be naturally achieved for solution to equation with interaction we propose to consider the following construction. Let $\xi : \mathbb{R}^d \to \mathbb{R}$ be a Gaussian random field with independent and identically distributed coordinates which have zero mean and covariance

$$E\xi_1(u)\xi_1(v) = \exp\{-\frac{1}{2}\|u-v\|^2\}.$$

The random field ξ has infinitely smooth trajectories. For probability measure μ on $\mathbb{R}^d$ denote by $\xi(\mu)$ its image $\xi(\mu) := \mu \circ \xi^{-1}$. Construct the coefficients for equation with interaction as follows

$$a(u,\mu) = E_\xi \alpha(u,\xi(\mu)), \ b(u,\mu,p) = E_\xi \beta(u,\xi(\mu),p)$$

for α and β being functionals from ξ. With such coefficients equation (1) can be treated as the lifting of initial equation to the space of functionals from ξ. In the talk we present conditions for existence of such solution $\{\mu_t; t \geq 0\}$ that the process $\{\xi(\mu_t); t \geq 0\}$ is stationary. Here we suppose that ξ and W are independent.

As an application of the general construction we consider the case when initial measure μ_0 is a visitation measure of a certain smooth closed curve (knot) γ_0. It is proved [4], that $\xi(\gamma_0)$ has no self-intersections a.s. For stationary random knot $\{\xi(\gamma_t); t \geq 0\}$ we present overview of its properties and results of modelling.

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Intermittency Phenomena for Mass Distributions of Stochastic Flows with Interaction

Andrey Dorogovtsev, Alexander Weiss

The intermittency phenomenon occurs when high peaks in a qunatity occur rarely but are signicant enough to influence the asymptotics of the underlying quantity. Mathematically speaking this can be characterised with moments. In this talk the topic of intermittency will be covered for mass distributions for stochastic differential equations with interaction, namely

$$\begin{cases} dx(u,t) & & = a(x(u,t),\mu_t)\mathrm{d}t + b(x(u,t),\mu_t)\mathrm{d}B_t \\ \forall u \in \mathbb{R}^d & x(u,0) & = u \\ \mu_t & & = \mu_0 \circ x^{-1}(\cdot,t) \end{cases}$$

where μ_0 is a probability measure and $a : \mathbb{R}^d \times \mathfrak{M}(\mathbb{R}^d) \to \mathbb{R}^d$ and $b : \mathbb{R}^d \times \mathfrak{M}(\mathbb{R}^d) \to \mathbb{R}^{d\times d}$ are coefficients. Here $\mathfrak{M}(\mathbb{R}^d)$ is the space of all probability measures on $\mathbb{R}^d$ and $d \geq 1$ denotes the dimension. In the talk only such measures will be investigated which possess a Lebesgue density. The space of probability measures will be equipped with the Wasserstein distance

$$\gamma(\mu,\nu) = \inf_{\kappa\in C(\mu,\nu)} \int\int \frac{|u-v|}{1+|u-v|}\kappa(\mathrm{d}u,\mathrm{d}v) \tag{1}$$

Theorem 1. *Let a and b be Lipschitz continuous with respect to all arguments and continuously differentiable with respect to the spatial variable. Moreover assume that a and b are bounded. Then μ_t is almost surely absoulutely continuous with respect to the d-dimensional Lebesgue measure for all $t \geq 0$ with Lebesgue density:*

$$p_t = p_0(x^{-1}(\cdot,t)) \det(Dx^{-1}(\cdot,t))$$

Then intermittency is defined as followed

Definition 1. $(p_t)_{t\geq 0}$ is intermittent if $(\frac{\lambda_p}{p})_{p\geq 1}$ is strictly increasing for

$$\lambda_p = \lim_{t\to\infty} \frac{\ln\left(\int_{\mathbb{R}^d} p_t^p(u)\mathrm{d}u\right)}{t}$$

It turns out intermittency exists under dissipativity conditions on the coefficients

Theorem 2. *Let a and b suffice the conditions of Theorem 1, assume furthermore*

(1) *There exists a continuously differentiable function ϕ such that ϕ and its derivative is bounded. Furthermore ϕ suffices for all $u, v \in \mathbb{R}^d$*

$$(u-v, \phi(u)-\phi(v)) \leq -\alpha|u-v|^2$$

for some $\alpha > 0$.

(2) *Let B be the Lipschitz constant of b with respect to the spatial variable. Then*

$$2\alpha - B^2(2q-1) > 0$$

where $q > d$

Then p_t is intermittent.

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Asymptotic distributions of terms of a variation series in the case of random sample size

A. A. Dzhamirzaev, I.N. Mamurov

The variation series is the starting point for many applied problems; this concept is widely used in issues of mathematical statistics and other fields of knowledge. Therefore, a larger number of publications are devoted to the study of the distributions of terms of a variation series (t.v.s.).

In classical mathematical statistics, as well as in the studies conducted by the above authors, the sample size from which a variation series is formed is considered deterministic. In this paper, the authors study the asymptotic distributions of t.v.s. in the case when the sample size itself is a random variable (r.v.), i.e. the characteristics of the general population under consideration are observed (due to certain circumstances) in a random number of tests. This situation often occurs in practice and is more general than the deterministic case, when the number of observations is considered non-random. Random sample size appears in statistical problems in reliability theory, queuing theory, sequential analysis, etc.

All available publications concerning random sample size can be divided into two groups. The first group includes studies, in which an essential condition is the independence of the random sample size from the observed values ("independent scheme"). The second group consists of studies, in which such a condition is not assumed ("dependent scheme"). In most of the works of the first group, regarding the sample size ν_n, it is assumed that $\frac{\nu_n}{n}$ as $n \to \infty$ converges in distribution to some r.v. While the studies of the second group are characterized by a stronger condition: convergence in probability $\frac{\nu_n}{n}$ as $n \to \infty$ to some positive r.v. [1], [2].

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Asymptetic estimates for a small parameter in Hartmann–Wintner law of the iterated logarithm

M. U.Gafurov

The work is devoted to further refinement of the classical Hartman-Wintner theorem on the law of the iterated logarithm for a sequence of independent, identically distributed random variables. Namely, an estimate of the rate of convergence in the form of convergent series of weighted probabilities of large deviations is established the exact asymptotes in the small parameter of the series, which is a refinement of the corresponding result [1]. Analogs of the obtained results were proved for a family of independent, identically distributed random variables indexed on sectors of the d-dimensional lattice of the Euclidean space. Regarding the concept of a sector, we recommend to have a look at the monograph [2].

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Limit Gibbs measures for 1-D lattice models with competing interactions

N.N.Ganikhodjaev

An Ising model with competing nearest-neighbour and next-nearest-neighbour interactions is considered on a Cayley tree of first order. Such model on the Cayley tree of second order was considered by Vannimenus [1]. The author was able to find new modulated limit Gibbs measures, in addition to the expected paramagnetic and ferromagnetic ones. These new limit Gibbs measures (phases) consist in a period-four phase and commensurate modulated phases. We show that for model on the Cayley tree of first order its phase diagram contain the same paramagnetic, ferromagnetic, antiferromagnetic and modulated phases except period-four phase.

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Gaussian structure in coalescing stochastic flows

K.V. Hlyniana[1], *A. A. Dorogovtsev*

In this talk, we consider an ordered family of standard Brownian motions on the real line $\{x(u,\cdot),\ u \in \mathbb{R}\}$, in which any two particles move independently until they meet and after that coalesce and move together as one. It is known that for any time $t > 0$, the set $x(\mathbb{R}, t)$ is almost surely locally finite, so the point measure $N_t([u_1, u_2]) := \#\ \{x(\mathbb{R}, t) \cap [u_1, u_2]\}$ is well-defined. We investigate linear functionals which are represented as integrals with respect to the point process N_t. A limit theorem with respect to the spatial variable is obtained for such integrals. Denote by $X_t^n(f) := \frac{1}{\sqrt{n}} \int_0^n f(u) N_t(du);\ t \in [t_0; T];\ n \geq 1.$

Theorem 1. *For any* $0 < t_1 < t_2 < \ldots < t_m < T$ *the following weak convergence holds* $\left(X_{t_1}^n(f), \ldots, X_{t_m}^n(f)\right) \Rightarrow (\zeta_{t_1}(f), \ldots \zeta_{t_m}(f))$, *as* $n \to \infty$, *where* $(\zeta_{t_1}(f), \ldots \zeta_{t_m}(f))$ *is a Gaussian vector. Moreover, if* f *is a 1-periodic Lipschitz function then the limiting Gaussian process* $\zeta_t(f), t \in [t_0, T]$ $(t_0 > 0)$ *has continuous modification.*

Considering the family $\{\zeta_f,\ f \in L_2([0,1])\}$ as a generalized Gaussian element, we use it to obtain a limit theorem for the multiple integrals with respect to the point process N_t.

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Operators of second quantization for Bernoulli noise

Anastasiia Hrabovets

In this work, we will construct an approximation of Gaussian white noise based on the sequence of Bernoulli random variables. Such normalized sums of independent Bernoulli random variables were studied in [1], [2]. We will develop the chaotic representation for functionals from Bernoulli noise similar to the Gaussian case [3]. In this work, operators of second quantization and Ornstein-Uhlenbeck semigroup operators for Bernoulli noise will be presented. We will construct a measure-valued Markov process associated with the Ornstein-Uhlenbeck semigroup and investigate the limit behavior of these measures.

Let $\{\varepsilon_n, n \geq 1\}$ be a sequence of independent random variables with Bernoulli distribution: $\mathbb{P}(\varepsilon_n = 1) = \mathbb{P}(\varepsilon_n = -1) = \frac{1}{2}$.

Define

$$\forall f \in C\left([0,1]\right): \ \varphi\left(f\right) := \sum_{k=1}^{n} f\left(\frac{k}{n}\right) \frac{\varepsilon_k}{\sqrt{n}}$$

Definition 1 . We will call a set $\{\varphi(f) : f \in C\left([0,1]\right)\}$ of random variables by the Bernoulli noise in $C\left([0,1]\right)$.

Theorem 1 (Weak convergence to Gaussian white noise)**.** *For any* $f_1, \ldots, f_n \in C([0,1])$:

$$(\varphi(f_1), \ldots, \varphi(f_k)) \longrightarrow \left((f_1, \xi), \ldots, (f_k, \xi)\right), \ n \to \infty,$$

where (f_i, ξ) *is an element of Gaussian white noise.*

Define the polynomials

$$A_k^n\left(\overrightarrow{\varepsilon}\right) = \frac{1}{n^{\frac{k}{2}}} \sum_{i_1 \neq \ldots \neq i_n} f\left(\frac{i_1}{n}, \ldots, \frac{i_k}{n}\right) \varepsilon_{i_1} \ldots \varepsilon_{i_k}, \ 1 \leq i_j \leq n.$$

Lemma 1. $\{A_k^n\left(\overrightarrow{\varepsilon}\right), 1 \leq k \leq n\}$ *is a system of orthogonal polynomials.*

Theorem 2. $\forall k \geq 1$:

$$A_k^n\left(\overrightarrow{\varepsilon}\right) = \frac{1}{n^{\frac{k}{2}}} \sum_{i_1 \neq \ldots \neq i_n} f\left(\frac{i_1}{n}, \ldots, \frac{i_k}{n}\right) \varepsilon_{i_1} \ldots \varepsilon_{i_k} \Longrightarrow f(\underbrace{\xi, \ldots, \xi}_{k}), \ n \to \infty,$$

where $f(\underbrace{\xi, \ldots, \xi}_{k})$ *is Hermite polynomial related to function* f *from definition for* $A_k^n\left(\overrightarrow{\varepsilon}\right)$.

Lemma 2. *Any function from $\{\varepsilon_n\}$ can be represented as*

$$\alpha(\overrightarrow{\varepsilon}) = \sum_{k=0}^{\infty} \sum_{i_1 \neq \ldots \neq i_k}^{n} a_{i_1 \ldots i_k} \varepsilon_{i_1} \ldots \varepsilon_{i_k}$$

Definition 2 . Define the operator of second quantization corresponding to $\{p_n, n \geq 1\}, p_n \in (-1,1)$ as follows

$$\Gamma(p)\alpha = \alpha(p\varepsilon) = \sum_{k=0}^{\infty} \sum_{i_1 \neq \ldots \neq i_k}^{n} a_{i_1 \ldots i_k} p_{i_1} \varepsilon_{i_1} \ldots p_{i_k} \varepsilon_{i_k}.$$

Lemma 3. *For any nonnegative α and any $\{p_n, n \geq 1\}, p_n \in (-1,1)$:*

$$\Gamma(p)\alpha \geq 0.$$

Consider analogs of operators of the Ornstein-Uhlenbeck semigroup for Bernoulli noise. For any $n \geq 1$ consider $p_n = e^{-t}$. Then

$$T_t^{\varepsilon}\alpha = \Gamma(p)\alpha = \sum_{k=0}^{\infty} \sum_{i_1 \neq \ldots \neq i_k}^{n} a_{i_1 \ldots i_k} e^{-kt} \varepsilon_{i_1} \ldots \varepsilon_{i_k}.$$

Consider a space

$$K = \{-1;1\}^{\mathbb{N}},\ \rho((x_n),(y_n))) = \sum_{n=1}^{\infty} \frac{1}{2^n} |x_n - y_n|$$

$\mathcal{M}^{\infty}$ is a set of all product measures on K :

$$\mathcal{M}^{\infty} \ni \mu = \mu_1 \otimes \mu_2 \otimes \ldots$$

Let $\{\tau_n\}$ be a sequence of independent exponentially distributed random variables with $\lambda = 1$.

For $\mu \in \mathcal{M}^{\infty}$ define the process $\{\mu_t\}$:

$$\mu_t = \mu_1^t \otimes \mu_2^t \otimes \ldots$$

where $\mu_n^t = \mu_n$ if $\tau_n > t$ and $\mu_n^t = \frac{1}{2}\delta_{-1} + \frac{1}{2}\delta_{+1}$ if $\tau_n \leq t$.

Consider $f : K \longrightarrow \mathbb{R}$

$$f(x) = \sum_{n=0}^{\infty} \sum_{1 \leq i_1 < \ldots < i_n} a_{i_n,\ldots,i_n} x_{i_1} \ldots x_{i_n}.$$

$$< f, \mu_t > = \int_K f(x) \mu_t(dx)$$

The element $x \in K$ can be identified with the measure

$$\delta_x = \delta_{x_1} \otimes \delta_{x_2} \otimes \dots$$

Then

$$\mathbb{E}_{\delta_x} < f, \mu_t >= \sum_{n=0}^{\infty} \sum_{1 \leq i_1 < \ldots < i_n} a_{i_n,\ldots,i_n} e^{-nt} x_{i_1} \ldots x_{i_n}$$

Properties:

- $\mu_t \longrightarrow \left(\frac{1}{2}\delta_{-1} + \frac{1}{2}\delta_{+1}\right)^{\infty}$ in Wasserstein metric
- Consider $f : \{-1, +1\}^{\infty} \longrightarrow \mathbb{R}$,

  $$f(x) = \sum_{k=1}^{\infty} \frac{x^k}{2^{k+1}}.$$

  Then

  $$\mu_t \circ f^{-1} \longrightarrow U\left[-\frac{1}{2}, \frac{1}{2}\right]$$

  in Wasserstein metric
- Consider

  $$g_n(x) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} x_k,$$

  then

  $$\mu_t \circ g_n^{-1} \longrightarrow N(0, 1).$$

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A family of F- quadratic stochastic operators

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Let $E = \{1, \ldots, m\}$ be a finite set and the set of all probability distributions on E

$$S^{m-1} = \{\mathbf{x} = (x_1, x_2, \ldots, x_m) \in \mathbb{R}^m : \ x_i \geq 0, \text{ for any } i \text{ and } \sum_{i=1}^{m} x_i = 1\},$$

be the $(m-1)$-dimensional simplex.

A *quadratic stochastic operator* (QSO) is a mapping $V\colon S^{m-1} \to S^{m-1}$ of the simplex into itself, of the form $V(\mathbf{x}) = \mathbf{x}' \in S^{m-1}$, where

$$(2) \qquad V : x'_k = \sum_{i,j=1}^{m} P_{ij,k} x_i x_j, \quad k = 1, \ldots, m$$

and the coefficients $\{P_{ij,k}\}$ satisfy

$$(3) \qquad P_{ij,k} = P_{ji,k} \geq 0, \quad \sum_{k=1}^{m} P_{ij,k} = 1, \quad i, j, k \in E.$$

The trajectory $\{\mathbf{x}^{(n)}\}_{n=0}^{\infty}$, of V for any point $\mathbf{x}^{(0)} \in S^{m-1}$ is defined by $\mathbf{x}^{(n+1)} = V\big(\mathbf{x}^{(n)}\big) = V^{n+1}\big(\mathbf{x}^{(0)}\big)$, $n = 0, 1, 2, \ldots$ One of the main goal of mathematical biology is to study of the asymptotic behaviour of the trajectories for a given QSO (see [1]).

Let us extend the set E by adding element "0", that is we consider $E_0 = \{0, 1, \ldots, m\}$. Fix a set $F \subset E$ and call this set the set of "females" and the set $M = E \setminus F$ is called the set of "males". The element "0" will play the role of "empty-body". Coefficients $P_{ij,k}$ of the matrix $\mathbf{P}$ we define as follows

$$(4) \qquad P_{ij,k} = \begin{cases} 1, & \text{if } \ k = 0, i, j \in F \cup \{0\} \ \text{ or } \ i, j \in M \cup \{0\}; \\ 0, & \text{if } \ k \neq 0, i, j \in F \cup \{0\} \ \text{ or } \ i, j \in M \cup \{0\}; \\ \geq 0, & \text{if } \ i \in F, j \in M, \forall k. \end{cases}$$

Definition 1.[2] For any fixed $F \subset E$, the QSO defined by (2),(3) and (4) is called $F-$ quadratic stochastic operator.

Let π_s be a permutation of the set $E = \{1, 2, \ldots, m\}$ and $\psi_k : E \to E$ be a map for any natural k. Consider $E_0 = \{0, 1, \ldots, m\}$ and

$$F_s = \{\pi_s(1), \pi_s(2), \pi_s(3), \ldots, \pi_s(\psi_s(m_1))\}, \quad M_s = \{\pi_s(\psi_s(m_1 + 1)), \ldots, \pi_s(m)\}.$$

Then we have that corresponding F_s- QSO with the matrix $\mathbf{P}^{\pi_s} = \left(P^{\pi_s}_{ij,k}\right)_{i,j,k\in E_0}$ has the form

$$(5) \qquad V_{\pi_s}: \begin{cases} x_0' = 1 - 2\sum\limits_{i\in F_s}\sum\limits_{j\in M_s}\left(1 - P^{\pi_s}_{ij,0}\right)x_ix_j; \\ x'_{\pi_{s+1}(k)} = 2\sum\limits_{i\in F_s}\sum\limits_{j\in M_s} P^{\pi_s}_{ij,\pi_s(k)}x_jx_j, \quad k = 1,2,\dots,m, \end{cases}$$

where the coefficients $P^{\pi_s}_{ij,k}$ satisfy the conditions

$$(6) \qquad P^{\pi_s}_{ij,\pi_s(k)} = P^{\pi_s}_{ji,\pi_s(k)} \geq 0, \quad k\in E_0; \quad \sum_{k=0}^{m} P^{\pi_s}_{ij,\pi_s(k)} = 1, \quad \forall i\in F_s, j\in M_s, \quad s>0.$$

Let $\mathcal{V}_\pi = \{V_{\pi_s}, s = \overline{1,m!}\}$ be the set of all F- QSOs defined on S^m. As any QSO V_{π_s} is represented by a matrix $\mathbf{P}^{\pi_s} = \left(p^{\pi_s}_{ij,k}\right)_{i,j,k\in E_0}$, the set $\mathcal{V}_\pi$ is compactly embedded in $\mathbb{R}^{(m+1)^3}$. Let $\mathcal{H}$ be the Borel σ-algebra induced on the set $\mathcal{V}_\pi$. Let $(\Omega, \mathfrak{F}, \mathbb{P})$ a probability space. Any measurable map $G:\Omega\to\mathcal{V}_\pi$ (i.e. such that $G^{-1}(\mathcal{H})\subset\mathfrak{F}$) is called a *random F- quadratic stochastic operator*. Consider the set $\mathcal{V}_\pi$ of measurable QSOs on S^m into S^m, with V_{π_s} assigned a positive probability q_s, $s=\overline{1,m!}$, $q_1+\cdots+q_{m!}=1$. Given an initial state $\mathbf{x}^{(0)}\in S^m$, one picks a F-QSO at random from the set $\mathcal{V}_\pi$, V_{π_s} being picked with probability q_s, $s=\overline{1,m!}$. Consider a random dynamical system $\mathbf{x}^{(n+1)} = T_{n+1}T_n\cdots T_1\mathbf{x}^{(0)} = T_{n+1}\left(\mathbf{x}^{(n)}\right) \quad (n\geq 1)$, where $\{T_n : n\geq 1\}$ is a sequence of independent F-QSOs on S^m into S^m with the common distribution $Q=\{q_1,\dots,q_{m!}\}$ on $\mathcal{V}_\pi$.

Theorem 1. *Let $Q=\{q_1,\dots,q_{m!}\}$ be common distribution on the set $\mathcal{V}_\pi$. Then*

$$\mathbb{P}\left(\lim_{n\to\infty} T_n(\mathbf{x}) = (1,\underbrace{0,\dots,0}_{m})\right) = 1.$$

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Weak dependence properties of a vertex process of a convex hull generated by a Poisson point process inside a parabola

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The convex hull generated by the implementation of a Inhomogeneous Poisson point process inside a parabola is studied in this article. The measure of intensity of the Poisson law is associated with regularly varying functions near the boundary of the carrier. It is worth noting that P. Groeneboom [1] was the first, who approximated a homogeneous binomial point process by the Poisson process, and using martingale properties for stationary vertex processes, proved the central limit theorem for the number of vertices of a convex hull in the case when the carrier of the uniform distribution is either a convex polygon or an ellipse. After [1], numerous studies appeared in which, developing the P. Groeneboom technique, various versions of central limit theorems were proved for the main functionals of a random convex hull for the case when the carrier of the original distribution is concentrated both in convex bounded and unbounded domains on the plane. The approach we used to study the properties of vertex processes differs from the P. Groeneboom technique and is based on analytical and direct probabilistic techniques. The main tool of the study is the independence properties of the increment of a Poisson point process, with which the area limited by the perimeters of the convex hull and the boundaries of the parabola carrier is represented as a sum of independent identically distributed random variables. Moreover, these quantities do not depend on the vertices of the convex hull (see, for example, [2]).

Let us denote the smallest root of the equation by b_n :

$$nx^{-\left(\beta+\frac{1}{2}\right)}L(x) = 1,$$

where $L(x)$ is the slowly varying function in sense of Karamata representable in the following form:

$$L(u) = \exp\left\{\int_1^u \frac{\varepsilon(t)}{t}dt\right\}, \quad \varepsilon(t) \to 0, \quad t \to \infty.$$

We assume that

$$R_n = \left\{(x,y):\ \frac{x^2}{2b_n} \le y\right\} \subset \mathrm{R}^2.$$

And denote a Inhomogeneous Poisson point process (i.h.p.p.) with intensity $\Lambda_n(\cdot)$ by $\Pi_n(\cdot)$, where the intensity measure $\Lambda_n(\cdot)$ is related the s.v.f. $L(x)$.

Let $(X_1, Y_1), (X_2, Y_2), ...$ be realizations of i.h.p.p. $\Pi_n(\cdot)$ in R_n, C_n are the convex hulls generated by these random points and Z_n are their set of vertices.

Assume that $e_0 = (0, 1)$.

We denote one of the vertices for which $(e_0, z - z_0) \geq 0$ for all $z \in Z_n$ by $z_0 \in Z_n$. It is obvious that z_0 is determined uniquely and almost certainly.

In this case, the straight line $(e_0, z - z_0) = 0$ is the line of support for C_n.

Let us now number vertices C_n, going around the boundary counterclockwise. Since z_0 has already been defined, each of the vertices thereby receives its own number $j, -\infty < j < \infty$. Then, from the condition of independence of the increment of the Poisson point process, the area bounded by perimeters C_n and parabola $v = \frac{u^2}{2b_n}$ is expressed as a sum of independent random variables $\xi_j, -\infty < j < \infty$. From here, it is possible to obtain the exact distribution $z_0 = (u_0, v_0)$ and distribution z_{j+1} subject to vertices z_{j-1} and z_j

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Limit theorems for reduced processes starting with a large number of particles

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Let $\{Z_k, k \geq 0\}$ the Galton-Watson branching process (see e.g.,[1]) in which the number of direct descendants of one particle have the generating function $f(s)$, $0 \leq s \leq 1$.

Denote by $Z(m,n)$ the number of particles in the moment $m(m \leq n)$ in the process $\{Z_k,\, k \geq 0\}$, whose descendants exit at the moment n. The random process $\{Z(m,n),\, 0 \leq m \leq n\}$ is called the reduced process generated by the process $\{Z_k,\, k \geq 0\}$. The reduced process $\{Z(m,n),\, 0 \leq m \leq n\}$ is called subcritical, critical and supercritical if $f'(1) < 1$, $f'(1) = 1$ and $f'(1) > 1$ respectively. Reduced subcritical processes for Galton-Watson processes were introduced by Fleischmann and Prehn [2]. Fleischmann and Sigmund-Schultze [3] proved a functional limit theorem (under the assumption $Z_n > 0$) in which.

The convergence of reduced critical processes to the Yule process is established. Liu and Vatutin[4] proved conditional limit theorems (under the assumption $0 < Z_0 \leq \psi(n)$) for rudused critical processes starting with a single particle and with a finite variance in the number of direct descendants of a single particle.

In this report, we propose limit theorems for subcritical and critical reduced processes $Z(m,n)$, $0 \leq m \leq n$ in the case when $Z_0 = \varphi(n)$ with probability 1, where $\varphi(n)$ such that $\varphi(n) \sim n$ or $\varphi(n) = o(n)$ when $n \to \infty$.

Here are some of our results.

Theorem 1. *Let for a reduced critical process $0 < f'(1) = \sigma^2 < \infty$ and with probability 1 $Z_0 = [xn]$, where is $x > 0$ a fixed number, the sign $[a]$ means the integer part of the number a. Then for any $t \in [0,1)$ the next relation holds*

$$\lim_{n\to\infty} E[s^{Z([nt]n)}/Z(0) = [xn],\, Z(n) > 0] = \frac{e^{-\frac{2x}{\sigma^2}\frac{1-s}{1-ts}} - e^{-\frac{2x}{\sigma^2}}}{1 - e^{-\frac{2x}{\sigma^2}}}$$

Theorem 2. *Let the conditions $A = f'(1) = E\xi < 1$, $\sum\limits_{k=2}^{\infty} p_k k \log k < \infty$ be satisfied for the branching process $\{Z(k),\, k \geq 0\}$ and with probability 1 $Z(0) = \psi(n) = [cA^{-n}]$, where $c > 0$ is constant. Then for any fixed $m \in N \cup 0$ the following asymptotic* *relations hold:*

$$E[s^{Z(n-m,n)}/Z(0)=\psi(n)] \to e^{cKA^{-m}(1-g(\varphi_m(s)))} \quad as \ n\to\infty,$$

$$E[s^{Z(n-m,n)}/Z(0)=\psi(n),\, Z(n)>0] \to \frac{e^{cKA^{-m}(1-g(\varphi_m(s)))}-e^{-ck}}{1-e^{-cK}} \quad as \ n\to\infty,$$

where

$$g(s)=\sum_{k=1}^{\infty} b_k s^k,\, b_k=\lim_{n\to\infty} P(Z(n)=k/Z(0)=1,\, Z(n)>0),$$

the number K is determined by the relation

$$K=(g'(1))^{-1},$$

$$\varphi_m(s)=f_m(0)+(1-f_m(0))s.$$

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The asymptotic of the probability of falling into zero of a multi-type branching process with immigration

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Consider the following two-stage model of branching processes with immigration. The process of reproduction with continuous time with a generating function

$$f_i(s) = \sum_{k\geq 0} f_k^i s^k, \quad \sum_{k\geq 0} f_k^i = 1, i = 1, 2$$

(the probability of incrementing one particle i- of that type per time $t \to 0$ is equal to $f_k^i t + o(t)$ at $k > 0$ and $1 + f_0^i + o(t)$ at $k = 0$) and the process of immigration with discrete time is given by the generating function

$$G^i(s) = \sum_{k\geq 0} G_k^i s^k, \quad \sum_{k\geq 0} G_k^i = 1, \quad G^i(0) = G_0^i > 0.$$

Definition: The lifetime of a branching process starting with n particles i- of that type with immigration has length τ if the number of particles $z_i(0) = n, z_i(t) > 0$ for all $t, 0 < t < \tau$, a $z_i(\tau) = 0$ (the trajectory of the process $z_i(t)$ is assumed to be continuous on the right). Suppose:

$\gamma_i(t) = P_i\{z_i(t) = 0/z_i(0) = 0\}$.

Theorem 1. *If $\theta_i = \lambda_i \mu_i b_i^{-1}, L(t)$ is a slow changer function and*

$$\sum_{k=1}^{\infty} k G_k^i Lnk < \infty, \sum_{k\geq 1}^{\infty} k^2 G_k^i Lnk < \infty,$$

then $\gamma \sim ct_i^{\theta}, (t \to \infty, c > 0)$, where $\lambda_i = \sum\limits_{k\geq 0} v_k^i S^k, \lambda_i = V_i^1(1) < \infty, \mu_i = \int_0^\infty = tdG_i(t)$.

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Asymptotic approximation for a certain class of statistics defined on several generalized urn models

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Urn models are a useful tool which allows to formulate and better understand many combinatorial problems in probability and statistics.Urn models naturally arise in statistical mechanics, clinical trials, cryptography etc. Properties of several types of urn models have been extensively studied in both probability and statistics literature; see e.g. survey paper by Kotz and Balakrishnan (1997). Most general definition of urn models has been introduced by Mirakhmedov et al (2014), where the asymptotic theory and higher-order expansions for a certain class of statistic are presented. Although their work covers many specific urn models it does not cover, such probabilistic models as, for instance, an infinite balls-in-boxes occupancy scheme , random allocation of particles in sets, and the statistics based on several samples from a population(s), intensively studied in the literature. Specifically, a number of recent publications associated with the infinite multinomial occupancy scheme, see e.g. Gnedin et al (2007), Gnedin and Iksanov (2020) and references therein, motivated this study. The generalized random allocation scheme we are interested here is as follows.

Let $\xi_k = (\xi_{k,1}, \xi_{k,2}, ...)$, $k = 1, ..., s$, be a collection of independent sequences (random vectors (r.vec)) of independent non-negative integer random variables (r.v.s) such that for each k the series $\zeta_k = \xi_{k,1} + \xi_{k,2} + ...$ is a.s. converges and $Pr\{\zeta_k = n_k\} > 0$ for a given integer $n_k \geq 2$. We consider the conditional distribution $\mathcal{L}(\xi_k \mid \zeta_k = n_k)$ which in turn generates a r.vec. $\eta_k = (\eta_{k,1}, \eta_{k,2}, ...)$ such that

$$\mathcal{L}(\eta_k) = \mathcal{L}(\xi_k \mid \zeta_k = n_k). \tag{7}$$

where $k = 1, ..., s$, and $\mathcal{L}(X)$ stands for the distribution of the r.vec. X. This equality implies that the integer $\eta_{k,m} \geq 0$, $Pr\{\eta_{k,1} + \eta_{k,2} + ... = n_k\} = 1$, and hence η_k should be viewed as r.vec. of frequencies in an urn model, where a sample of size n_k is drawn from an urn containing a finite or infinitely many types of items labelled by $\{1, 2, ...\}$, then the r.v. η_{km} becomes as number of items of m-th type in the sample. The sample scheme determines by the distributions of r.v.s $\xi_{k,m}$ and parameter n_k. The probabilistic model defined by (1) is what we call a "generalized urn model" (GUM). Thus we deal with s independent GUMs. An alternate interpretation of the model (1) is the random allocation of n_k particles into cells labeled by $\{1, 2, ...\}$. The r.v. $\eta_{l,m}(n_l)$ then is the number of particles falling into the m-th cell after allocation of all n_k particles. The independent r.vec.s $\eta_1, ..., \eta_s$ all together can be viewed, for example, as a random allocation of particles of s types in the infinite multinomial occupancy scheme , where n_k is the number of particles of the k- th type, as well as random allocation of particles in sets, where now n_k is the number of particles of k-th set.

Our aim here is to present a unified approach to the derivation of an asymptotic (as $min(n_1, ..., n_s) \to \infty$) approximation for the distribution function of statistics of the form

$$R(n) = \sum_{m=1}^{\infty} f_m(\eta_{1,m}, ..., \eta_{s,m}), \tag{8}$$

where $f_m(x_1, ..., x_s)$, $m = 1, 2, ...$,is a sequence of functions (may be random) defined for $x_1 \geq 0, ..., x_s \geq 0$, such that the series (2) is a.s. converges for every $n = (n_1, ..., n_s)$. The generalized urn model and statistics considered by Mirakhmedov et al (2014) follows if $s = 1$ and $Pr\{\xi_{1,m}(n_1) = 0\} = 1$ for $m > N$. some $N = N(n) \to \infty$.

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Limit theorems for U-statistics of positively associated random variables

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Limit theorems for $U-$statistics of weakly dependent random variables in one and two samples cases were studied by many authors (see for example [1]-[4]).

We consider as a measure of dependency the following coefficients

$$r(k) = \sup_{\substack{x,y \in R \\ i \in N}} \left[P(X_i > x, X_{i+k} > y) - P(X_i > x) P(X_{i+k} > y) \right], \ k \in N.$$

Define the associated random variables.

Definition 1. A sequence of random variables $\{X_n, \ n \geq 1\}$ is said to be positively associated, if for any finite collection $\{X_{i_1}, .., X_{i_n}\}$ and any real coordinatewice non-decreasing functions f, g on R^n

$$Cov\left(f\left(X_1, ..., X_n\right), g\left(X_1, ..., X_n\right)\right) \geq 0,$$

whenever the covariance is defined.

For a stationary sequence $\{X_n, \ n \geq 1\}$ with common distribution function F, $U-$statistics are defined as following

$$U_n = \frac{2}{n(n-1)} \sum_{i<j} h(X_i, X_j)$$

where $h(x,y)$ is a measurable symmetric function. According Hoeffding decomposition we have:

$$h(x,y) = \theta + h_1(x) + h_2(y) + g(x,y),$$

where $h_1(x) = Eh(x,Y)$, $h_2(y) = Eh(X,y)$, $g(x,y) = h(x,y) - h_1(x) - h_2(y) - \theta$.
$\theta(F) = \iint_{R^2} h(x_1, x_2)\, dF(x_1)\, dF(x_2)$.
Denote $\sigma_1^2 = Var(h_1(X_1))$, $\sigma_{1j}^2 = Cov(h_1(X_1); h_1(X_{1+j}))$, $\sigma_U^2 = \sigma_1^2 + 2\sum_{j=1}^{\infty} \sigma_{1j}^2$.
Assume that the following conditions hold:

$$\iint_{R^2} |h(X_1, X_2)|^s \, dF(x_1)\, dF(x_2) < C < \infty, \ s \geq 2 \text{ and}$$

$$E\left|g(X_1, X_j)\right|^s < C < \infty, \ s \geq 2, \ j \in N \tag{1}$$

Theorem 1. *Let $\{X_i, i \geq 1\}$ be a stationary sequence of positively associated random variables with marginal distribution function F. Assume that $h(x,y)$ is monotone in and there exists a positive δ such that for $s = 2+\delta$ the condition* (1) *and*

$$r(n) = O\left(n^{-\left(2+\delta^{/}\right)/\delta^{/}}\right), \text{ for some } \delta^{/} \ \left(0 < \delta^{/} < \delta\right).$$

hold. If $0 < \sigma_U^2 < \infty$, then the following weak convergence holds

$$\frac{\sqrt{n}}{2\sigma_U}\left(U_n - \theta(F)\right) \Rightarrow N(0,1),$$

where N(0,1) is a standard Gaussian random variable.

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Functional limit theorems for perturbed random walks

A.Y.Pilipenko [1]

Consider a random walk $S_\xi(n) := \xi_1 + \cdots + \xi_n$, where (ξ_k) are independent copies of an integer-valued random variable ξ, where $\mathrm{E}\xi = 0$ and $\sigma^2 := \mathrm{Var}\xi \in (0, \infty)$. Due to the Donsker invariance principle we have convergence in distribution

$$\frac{S_\xi([n\cdot])}{\sigma\sqrt{n}} \Rightarrow W(\cdot),\ n \to \infty,$$

in space $D([0, \infty))$.

Consider an integer-valued Markov chain X whose transition probabilities coincide with transition probabilities of S_ξ everywhere except of a finite set $A \subset \mathbb{Z}$. We will call the set A the membrane and we shall say that X is a random walk that is perturbed on A. It appears that the Donsker scaling of X may be not a Brownian motion but a singular diffusion. The first result on this topic belongs to Harrison and Shepp [2] who considered a perturbation of a simple symmetric random walk at $A = \{0\}$. They proved that if

$$\mathrm{P}(X(1) = x \pm 1 \mid X(0) = x) = \frac{1}{2},\ x \neq 0,$$

$$\mathrm{P}(X(1) = 1 \mid X(0) = 0) = p \in [0, 1],$$

$$\mathrm{P}(X(1) = -1 \mid X(0) = 0) = q := 1 - p,$$

then the process $\frac{X([n\cdot])}{\sqrt{n}}$ converges in distribution to a skew Brownian motion W_γ^{skew} with permeability parameter $\gamma = p - q$. Recall that W_γ^{skew} is a continuous homogeneous Markov process with transition probability density function

$$p_t(x, y) = \varphi_t(x - y) + \gamma \mathrm{sgn}(y)\varphi_t(|x| + |y|), \quad x, y \in \mathbb{R}, \quad t > 0,$$

where $\varphi_t(z) = (2\pi t)^{-1/2} e^{-\frac{z^2}{2t}}$.

Harrison and Shepp used the classical approach in their proof, they verified tightness and convergence of finite-dimensional distributions that can be calculated directly using a simple structure of transition probabilities and Andre's reflection principle. Naturally, the case of general membrane A and non-unit jump outside of A is

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much harder to deal with. Some partial results on functional limit theorems generalized result of [2] were obtained in [4, 6, 7, 3, 5], where the limit process was also a skew Brownian motion under some additional restrictions that were important during the corresponding proofs, say finiteness of jumps outside of the membrane, membrane consists of one point, etc.

We obtain a final result that uses a minimal set of assumptions and generalizes all previous works.

Consider an integer-valued time-homogeneous Markov chain $(X(k))_{k\geq 0}$ that behaves as a random walk everywhere, except of the finite set $\{-d,\dots,d\}$, $d\geq 0$, called a membrane. We will assume that if X is located to the right of the membrane, then its jumps have a distribution ξ_+, and its jumps to the left of the membrane have a distribution ξ_-, that is

$$\mathrm{P}(X(1)=x+y|X(0)=x)=\mathrm{P}(\xi_+=y),\quad x>d,$$

$$\mathrm{P}(X(1)=x+y|X(0)=x)=\mathrm{P}(\xi_-=y),\quad x<-d.$$

The laws of jumps from the membrane, i.e. $\mathrm{P}(X(1)\in\cdot|X(0)=x)$, $|x|\leq d$, are arbitrary.

Theorem 1. *Assume that* $\mathrm{E}\,\xi_\pm=0$, $v_\pm^2:=\mathrm{Var}\,\xi_\pm\in(0,\infty)$, *the states* $\mathbb{Z}\backslash\{-d,\dots,d\}$ *of Markov chain* X *are connected, and jumps from the membrane have finite expectation, i.e.*

$$\max_{|x|\leq d}\mathrm{E}\big[|X(1)\,\big|\,X(0)=x\big]<\infty.$$

Let

$$\varphi(x)=x(v_+^{-1}1\!\!\mathrm{I}_{x\geq 0}+v_-^{-1}1\!\!\mathrm{I}_{x<0}).$$

Then the Donsker scaling $\big\{\varphi\big(\frac{X([n\cdot])}{\sqrt{n}}\big)\big\}_{n\geq 1}$ *weakly converges to a skew Brownian motion* $W_\gamma^{\mathrm{skew}}(\cdot)$ *starting at 0 with some permeability parameter* $\gamma\in(-1,1)$.

Our methods are based on a martingale characterization of the skew and the Walsh Brownian motion. The approach was initiated in [3, 1]. We allow different variance on different sides of the membrane, which was not done in all previous works except of [5]. The case when the membrane A is an arbitrary finite set $\{x_1,\dots,x_m\}\subset\mathbb{Z}$ not necessarily equal to $\{-d,\dots,d\}$, is also covered by Theorem 1. Indeed, X can be considered as a random walk perturbed on bigger membrane $\{-d,\dots,d\}$, where $d=\max_{1\leq i\leq m}|x_i|$.

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Martingale representation of Brownian functionals

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We study functionals whose filter is not stochastically smooth and propose a method for finding the integrand. The question of representing Brownian functionals as a stochastic It integral with an explicit form of the integrand is investigated. The class of functionals under consideration also includes functionals that are not smooth in the sense of Malliavin, to which both the well-known Clark-Ocone formula (1984) and its generalization, the Glonti-Purtukhia representation (2017), are inapplicable.

Let a Brownian Motion $B = (B_t)$, $t \in [0,T]$, be given on a probability space $(\Omega, \Im, P)$, and let $\Im_t^B = \sigma\{B_u : 0 \le u \le t\}$.

Theorem 1. *Let $f(\cdot,\cdot) : [0,T] \times R^1 \longrightarrow R^1$ be a measurable bounded function, then the function $V(t,x) = E[\int_t^T f(s,B_s(\omega))ds | B_t = x]$ satisfies the requirements of the Ito formula and the the following stochastic integral representation is fulfilled*

$$\int_0^T f(s,B_s)ds = \int_0^T Ef(s,B_s)ds + \int_0^T V_x'(s,B_s)dB_s \quad (P-a.s.).$$

Corollary 1. *The following stochastic integral representation is valid*

$$I_{\{B_T \le c\}} = \Phi\Big(\frac{c}{\sqrt{T}}\Big) - \int_0^T \frac{1}{\sqrt{T-s}}\varphi\Big(\frac{c-B_s}{\sqrt{T-s}}\Big)dB_s \quad (P-a.s.),$$

where Φ is the standard normal distribution and φ is its density function.

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Weak periodic Gibbs measures for the Potts-SOS model on a Cayley tree of order two

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Potts-SOS model is generalization of the Potts and SOS (solid-on-solid) models. Weak periodic Gibbs measures for this model on the CT were not studied yet. In this work we study weak periodic Gibbs measures for this model on the CT of order two.

The Cayley tree τ^k (see [1]) of order $k \geq 1$ is an infinite tree, i.e. a graph without cycles, from each vertex of which exactly $k+1$ edges issue.

The Hamiltonian of the Potts-SOS model with nearest-neighbor interaction has the form

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} |\sigma(x) - \sigma(y)| - J_p \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)},$$

where $J, J_p \in R$ are nonzero coupling constants.

Let $H_A = \{x \in G_k : \sum_{i\in A} \omega_x(a_i)$ is an even number$\}$, where $\emptyset \neq A \subseteq N_k = \{1, 2, 3, ..., k+1\}$ and $\omega_x(a_i)$ is the number of letters a_i in a word $x \in G_k$. We get the following theorem:

Theorem 1. *Let $k = 2, \mid A \mid= 1, J_p = 2J$. Then all weak periodic Gibbs measures for the Potts-SOS model are translation-invariant.*

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Strong measurable continuous modification of the Burdzy-Kaspi stochastic flow

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Let $(\mathrm{W}_t)_{t\in\mathbb{R}}$ be a Brownian motion on $\mathbb{R}$. For all $(s,x)\in\mathbb{R}^2$ the following equation has a unique strong solution [1]:

$$X(t) = x + \mathrm{W}(t) - \mathrm{W}(s) + \beta \mathrm{L}(t),\ t \geq s, \tag{1}$$

where L is the symmetric local time of X at zero, and $\beta \in [-1,1]$. According to [2, 3] there exists a stochastic flow $\psi = (\psi_{s,t})_{-\infty<s\leq t<\infty}$ of measurable mappings of $\mathbb{R}$, such that for all $(s,x)\in\mathbb{R}^2$ $\psi_{s,\cdot}(x)$ is a solution to (1).

The stochastic flow ψ satisfies the evolutionary property in the sense that for all $r \leq s \leq t$ and $x \in \mathbb{R}$

$$\psi_{r,t}(x) = \psi_{s,t} \circ \psi_{r,s}(x) \text{ a.s.}$$

We will improve this result by showing that ψ has a modification $\tilde{\psi}$ such that $(s,t,x,\omega) \mapsto \tilde{\psi}_{s,t}(\omega,x)$ is measurable, and $\tilde{\psi}$ satisfies the evolutionary property for all r,s,t,x,ω simultaneously.

The proof will be based on the novel approach to constructing modifications of stochastic flows.

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An intersection of joint trajectories of independent Brownian motions in Carnot groups with a given set

O. Rudenko

Let G_i, $i = 1, \ldots, n$ be Carnot groups in $\mathbb{R}^d$. Denote as L_{ij}, $j = 1, 2 \ldots d$, $i = 1, 2, \ldots, n$ n sets of corresponding vector fields, that constitute the basis of the corresponding Lie algebra of left-invariant vector fields for each G_i, denote as p_{ij}, $j = 1, 2 \ldots d$, $i = 1, 2, \ldots, n$ the corresponding homogeneous degrees of L_{ij} and denote as $\rho_i, i = 1, 2, \ldots, n$ the corresponding distances on these groups. For each $i = 1, \ldots, n$ let X_i be a Brownian motion in G_i corresponding to $L_{ij}, j = 1, 2, \ldots, d$ (for details on all these definitions see [1]).

We suppose that X_i, $i = 1, \ldots, n$ are independent, and denote

$$Y(t) = (X_1(t_1), X_2(t_2), \ldots, X_n(t_n)), t = (t_1, \ldots, t_n)$$

. We want to find a condition when the following probability

$$P_{vis}(x, V) = P(\exists t \in (0, +\infty)^n : Y(t) \in V / Y(0) = x)$$

is zero, where $V \subset \mathbb{R}^{nd}$. For this purpose we use the following special definition of a Hausdorff measure on $\mathbb{R}^{nd}$. Additionally we can also obtain a bound on values of such Hausdorff measure, if the above probability is not zero.

For each $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^{nd}$, $y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^{nd}$ and positive numbers r_1, $r_2 \ldots r_n$, we denote $\gamma_{r_1,\ldots,r_n}(x, y) = \max\limits_{i=1,\ldots,n} \rho_i^{\frac{1}{r_i}}(x_i, y_i)$ and for $r \geqslant 0$ and $a > 0$ we denote $h_r(a) = a^r$.

Definition 1. For all $r \geqslant 0$ and positive r_1, $r_2 \ldots r_n$, we define a Hausdorff measure $H_{r,r_1,\ldots,r_n}$ in $\mathbb{R}^{nd}$ as follows:

$$H_{r,r_1,\ldots,r_n}(A) = \lim_{\varepsilon \to 0+} \inf\{\sum_{i=1}^{+\infty} h_r(\underset{\gamma_{r_1,\ldots,r_n}}{diam}(A_i)) | A \subset \bigcup_{i=1}^{+\infty} A_i, \underset{\gamma_{r_1,\ldots,r_n}}{diam}(A_i) < \varepsilon, i = 1, 2, \ldots\}$$

where $\underset{\gamma_{r_1,\ldots,r_n}}{diam}(A_i) = \sup\{\gamma_{r_1,\ldots,r_n}(x, y) | x \in A_i, y \in A_i\}$.

Let $B_i(y, \varepsilon)$ be an open ball with center y and radius ε with regard to the distance ρ_i and $Q_i = \sum\limits_{j=1}^{d} p_{ij}$.

Theorem 1. *Let V be a Borel set in $\mathbb{R}^{nd}$. Suppose that there exist a finite measure ν and positive constants C, $r_1, r_2 \ldots, r_n$ and $\delta \in \mathbb{R}$, such that for all $y = (y_1, y_2, \ldots, y_n) \in V$, $\varepsilon \in (0,1)$:*

$$\nu(B_1(y_1, \varepsilon^{r_1}) \times B_2(y_2, \varepsilon^{r_2}) \times \ldots \times B_n(y_n, \varepsilon^{r_n})) \geqslant C\varepsilon^{\sum\limits_{i=1}^{n} r_i(Q_i-2)+\delta} \tag{1}$$

If $\delta < 0$ then for all $x \in \mathbb{R}^{nd}$ we have $P_{vis}(x, V) = 0$.

If $\delta \geqslant 0$, then for all $x \in \mathbb{R}^{nd}$, such that

$$\int\limits_{\overline{V}} \prod_{i=1}^{n} \rho_i^{2-Q_i}(x_i, y_i)\nu(dy_1 \ldots dy_n) < +\infty \tag{2}$$

where $\overline{V}$ is a closure of V, and for all $s > 0$

$$P(H_{\delta, r_1, \ldots, r_n}(V \cap \{Y(t) | t \in [0, s]^n\}) < +\infty / Y(0) = x) = 1.$$

In the proof of this theorem several results about Carnot groups from [2] are used, as well as bounds on the densitiy of Brownian motion in Carnot group from [3]. Also we apply probabilistic potential theory developed in [4] to Brownian motion in Carnot group.

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Functional limit theorems for branching processes with non-stationary immigration

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Let $\{\xi_{k,i}, k, i \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ be two sequence of non-negative integer-valued random variables such that the two families $\{\xi_{k,i}, k, i \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ are independent, $\{\xi_{k,i}, k, i \geq 1\}$ are independent and identically distributed (i.i.d.). We consider a sequence of branching processes with immigration $X_k, k \geq 0$, defined by recursion:

$$X_0 = 0, \quad X_k = \sum_{i=1}^{X_{k-1}} \xi_{k,i} + \varepsilon_k, \quad k \geq 1. \tag{1}$$

Intuitively, one can interpret $\xi_{k,i}$ as the number of offsprings produced by the i-th individual belonging to the $(k-1)$-th generation and ε_k is the number of immigrants in the k-th generation. We can interpret X_k as the number of individuals in the k-th generation.

Assume that $a := \mathbb{E}\xi_{1,1} < \infty$. Process X_k is called subcritical, critical or supercritical depending on $a < 1$, $a = 1$ or $a > 1$, respectively. We refer the reader to recent survey of [1] where one can find a historical overview of limit theorems for process (1).

In the talk we will discuss the conditions which ensure the validity of functional limit theorems for critical process defined by (1) in the case when the immigration sequence $\{\varepsilon_k, k \geq 1\}$ is not necessarily identically distributed and generated by weakly dependent random variables. Our result extends the previous known result [2] in the literature.

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Central limit theorem for strong mixing random variables with values in $L_p[0,1]$ space

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Central limit theorems in Banach spaces are well studied in the case of independent identically distributed random elements (see [1]). Our goal is to establish a central limit theorem for strong mixing random variables with values in $L_p[0,1]$ space.

We say that a sequence $\{X_i(t), i \geq 1\}$ of centered random variables in $L_p[0,1]$ satisfies central limit theorem if the following weak convergence holds:

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i(t) \Rightarrow N(t)$$

where $N(t)$is some $L_p[0,1]$-valued Gaussian random variable with mean zero.

We will assume that $\{X_n(t), n \geq 1\}$satisfies α_m-mixing condition. For the sequence of $L_p[0,1]$-valued random variables $\{X_n(t), n \geq 1\}$ α_m-mixing coefficients are defined as:

$$\alpha_m(n) = \sup_{\Pi_m} \sup \left\{ |P(B) - P(A)P(B)| : A \in F_1^k(m), \quad B \in F_{k+n}^{\infty}(m), \quad k \in N \right\}$$

where $\Im_a^b(m)$-is σ-field generated by random variables $\prod_m X_a(t), ..., \prod_m X_b(t)$ and $\prod_m : L_p[0,1] \to R^m$ is projective operator i.e. $\prod_m X_i(t) = (X_i(t_1), ..., X_i(t_m))$, $t_i \in [0,1]$.

We say that $\{X_i(t), i \geq 1\}$ is α_m-mixing, if $\alpha_m(n) \to 0$ as $n \to \infty$, $m = 1, 2, ...$.

Our main result is the following

Theorem 1. *Let $\{X_i(t), i \geq 1\}$ be a strictly stationary sequences of random variables with values in $L_p[0,1]$, $1 < p \leq 2$ and for some $\delta > 0$*

$$EX_1(t) = 0$$

$$E\left|X_1(t)\right|^{2+\delta} < \infty$$

$$\sum_{n=1}^{\infty} (\alpha_m(n))^{\frac{\delta}{2+\delta}} < \infty, m = 1, 2, ...$$

$E\left|X_1(t+h) - X_1(t)\right|^{2+\delta} \leq f(h)$ *for* $0 \leq h < 1$, $0 \leq t \leq 1 - h$, *for some function* $f(\cdot)$*such that* $f(h) \to 0$ *as* $h \to 0$.

Then $\{X_i(t), i \geq 1\}$ satisfies a central limit theorem.

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Extremal properties of Bernoulli random variables

Sh. Shorakhmetov

In theoretical and practical problems of mathematics and other fields of science there are problems of finding extreme values of functionals . The article is devoted to finding the exact upper bounds of the functional $Ef(\xi_1 + \xi_2 + ... + \xi_n)$.

Let $F(x_1, x_2, ..., x_n)$ be the joint distribution of arbitrarily dependent random variables $\xi_i \in [0, 1]$, $i = \overline{1, n}$. Denote by $\mathcal{F}$ the class of all distributions with fixed mathematical expectations:

$$\mathcal{F} = \{F(x_1, x_2, ..., x_n) : \ E\xi_1 = m_1, ..., E\xi_n = m_n\}.$$

Without loss of generality, we assume that $m_1 \le m_2 \le ... \le m_n$.

Theorem 1. *Let f be the convex increasing function. Then*

$$\sup_{F \in \mathcal{F}} Ef(\xi_1 + \xi_2 + ... + \xi_n) = \sum_{k=0}^{n} f(n-k)(m_{k+1} - m_k),$$

where $m_0 = 0$, $m_n = 1$. The supremum is reached on binomial distributed random variables $B_1, B_2, ..., B_n$:

$$P(B_1 = 1, B_2 = 1, ..., B_n = 1) = m_1,$$

$$P(B_1 = 0, B_2 = 0, ..., B_n = 0) = 1 - m_n,$$

$$P(B_{i_1} = 0, ..., B_{i_l} = 0, B_{i_{l+1}} = 1, ..., B_{i_n} = 0) = m_{i_{l+1}} - m_{i_l}, l = 1, 2, ..., n-1.$$

The probability values of the remaining sets 0, 1 *equal to zero.*

A similar statement is true when the function is f monotonically decreasing.

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Operator splitting methods for non-homeomorphic one-dimensional stochastic flows

M.B. Vovchanskyi

We consider flows $\{X(\cdot,t) \mid t \geq 0\}$ of random transformation of the real line that represent the joint movement of interacting Brownian particles with Lipschitz continuous drift a whose pairwise correlation depends on the distance between them and is described in the terms of infinitesimal covariance φ. More precisely, for every u

$$dX(u,t) = a\left(X(u,t)\right)dt + dW_u(t), t \geq 0, \quad X(u,0) = u,$$

for some Wiener process W_u, and for every u, v

$$\frac{d}{dt}\left\langle X(u,t), X(v,t)\right\rangle(t) = \varphi\left(X(u,t) - X(v,t)\right), \quad t \geq 0.$$

Infinitesimal covariance φ is assumed to be either continuous or to equal $\mathbf{1}[x=0]$. The flow is known as the Brownian web and consists of particles that move independently before they meet in the latter case. In contrast to the case of diffeomorphic stochastic flows, mappings $\{X(u,\cdot) \mid u \in \mathbb{R}\}$ may be discontinuous, in which case clusters are formed within the flow.

The well-known operator splitting scheme is applied to the Harris flows introduced above so that the actions of the semigroups generated by the corresponding driftless Harris flow and the ordinary ODE $\frac{df(t)}{dt} = a(f(t))$ are separated.

We establish the weak convergence of finite-dimensional motions in Skorokhod spaces. This result is used to derive the convergence of the pushforward measures under the action of the corresponding flows for the Brownian web or under an additional assumption that guarantees the initial Harris flow to be a coalescing one. As another application, the convergence of the associated dual flows in reversed time is obtained. The rate of convergence is given for the Brownian web.

The talk is based on [1, 2, 3].

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On asymptotics of solutions of stochastic differential equations with jumps

V. K. Yuskovych

Let B be a Brownian motion, $\tilde{N}$ be a compensated Poisson random measure. We study the asymptotics of stochatic differential equations of the form

$$\text{(1)}\quad \mathrm{d}X(t) = a(X(t))\mathrm{d}t + b(X(t))\mathrm{d}B(t) + \int_{\mathbb{R}} c(X(t-), u)\tilde{N}(\mathrm{d}t, \mathrm{d}u),\ X(0) = x_0 \in \mathbb{R},$$

as $t \to \infty$ under the assumption that $X(t) \to +\infty$ almost surely.

The following theorem gives sufficient conditions of asymptotic equivalence of solution X and the solution of the ordinary differential equation

$$\mathrm{d}x(t) = Ax^{\alpha}(t)\mathrm{d}t,\ x(0) > 0.$$

Theorem 1. *Let X be a solution of stochastic differential equation (1) such that $X(t) \to +\infty$ a.s. and let $\alpha \in [0, 1)$. Suppose that the coefficients of equation (1) satisfy the conditions:*

- *for some constant $A_+ \geq 0$,*

$$|a(x)| \leq A_+ x^{\alpha}, \quad x \geq 1;$$

- *for some constant $A > 0$,*

$$a(x) \sim Ax^{\alpha},\ x \to +\infty;$$

- *for some constants $C \geq 0$ and $\beta \in \left[0, \frac{1+\alpha}{2}\right)$,*

$$b^2(x) + \int_{\mathbb{R}} c^2(x, u)\nu(\mathrm{d}u) \leq C\left(1 + |x|^{2\beta}\right), \quad x \in \mathbb{R}.$$

Then

$$X(t) \sim ((1-\alpha)At)^{\frac{1}{1-\alpha}}, \quad t \to \infty,\ \textit{a.s.}$$

Remark 1. We also study sufficient conditions such that $X(t) \to +\infty$ a.s.

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